Logic – Sample Questions

1. T/F:
   a. $\left(A \wedge B\right) \Rightarrow C \; entails \left(A \Rightarrow C\right) \vee \left(B \Rightarrow C\right)$ **True, actually equivalent – use truth tables or convert to CNF**
   b. $\left(P \wedge \neg R\right) \Rightarrow \left(Q \Rightarrow R\right)$ can be converted into a Horn clause. **True using logical equivalences:** $P \wedge Q \Rightarrow R$
   c. $\left(\forall x\ P(x)\right) \vee \left(\forall x\ \neg P(x)\right)$ is a valid sentence. **False *P* could be true sometimes but not always**
   d. $\forall x\ x = x$ is satisfiable. **True (also valid)**
2. Consider $\left(A \vee B\right) \wedge \left(\neg A \vee C\right) \wedge \left(\neg B \vee D\right) \wedge \left(\neg C \vee G\right) \wedge \left(\neg D \vee G\right)$
   Use resolution to prove that the sentence entails *G*.
   clauses 1 and 2 resolve to $\left(B \vee C\right)$ **this resolves with clause 4 to give** $\left(B \vee G\right)$ **this resolves with clause 3 to give** $\left(G \vee D\right)$ **which resolves with clause 5 to give, which resolves with the negated query to give the empty clause**
3. Correct each logic representation of the following sentences:
   a. "No two people have the same social security number"
      $\neg \exists x, y, n\ Person(x) \wedge Person(y) \Rightarrow (HasSS\#(x,n) \wedge HasSS\#(y,n))$
      incorrect – uses implication with existential. Correct is
      $\neg \exists x, y, n\ Person(x) \wedge Person(y) \wedge \neg\left(x = y\right) \wedge (HasSS\#(x,n) \wedge HasSS\#(y,n))$
   b. "John's social security number is the same as Mary's"
      $\exists n\ HasSS\#(John, n) \wedge HasSS\#(Mary, n)$
      correct
   c. "Everyone's social security number has 9 digits"
      $\forall x, n\ Person(x) \Rightarrow (HasSS\#(x,n) \wedge Digits(n,9))$
      Incorrect – says everyone has a number. Correct is:
      $\forall x, n \left(Person(x) \wedge HasSS\#(x,n)\right) \Rightarrow Digits(n,9)$
   d. Rewrite the above sentences (uncorrected) using the function symbols SS# instead of the predicate HasSS#.
      $\neg \exists x, y\ Person(x) \wedge Person(y) \Rightarrow SS\#(x) = SS\#(y)$
      $SS\#(John) = SS\#(Mary)$
      $\forall x\ Person(x) \Rightarrow Digits(SS\#(x), 9)$
4. Translate the following sentences into FOL using the predicates *French, Chilean, Wine, >,* and the functions *Price* and *Quality*:

a. All French wines cost more than Chilean wines.

$\forall x\ French(x) \wedge Wine(x) \Rightarrow (\forall y\ Chilean(y) \wedge Wine(y) \Rightarrow > (Price(x), Price(y)))$

b. The best Chilean wines are better than some French wines.

$\exists x\ Chilean(x) \wedge Wine(x) \wedge (\forall y\ Chilean(y) \wedge Wine(y) \Rightarrow \neg > (Quality(y), Quality(x))) \wedge (\exists z\ Frenc$

5. Show that the sentences:

a. $\forall x\ (\forall y\ P(x,y)) \Rightarrow Q(x)$

$= \forall x \neg (\forall y\ P(x,y)) \vee Q(x)$

$= \forall x (\exists y\ \neg P(x,y)) \vee Q(x)$

$= \neg P(x, f(x)) \vee Q(x)$

b. $\forall x\ \exists y\ (P(x,y) \Rightarrow Q(x))$

$= \forall x \exists y (\neg P(x,y) \vee Q(x))$

$= \neg P(x, f(x)) \vee Q(x)$

are logically equivalent by converting them to CNF. Give English sentences that interpret *P* and *Q* to make the sentences true in the real world.

***P* could be "hates" and *Q* could be "misanthrope", or *P* could be "loves" and *Q* could be "philanthrope"**

6. Assume the following propositions: *BatteryDead*, *RadioWorks, OutOfGas,* and *CarStarts*.

a. What is the total number of models? **2^4 = 16 models**

b. How many models are there in which the following sentence is false?

$(RadioWorks \wedge CarStarts) \Rightarrow (\neg OutOfGas \wedge \neg BatteryDead)$

$R \wedge C$ **is true in 4 models,** $\neg(\neg O \wedge \neg B) = O \vee B$ **is false in three of the four models**

c. Is the sentence above equivalent to a set of Horn clauses?

Yes, $R \wedge C \Rightarrow \neg O$ **and** $R \wedge C \Rightarrow \neg B$

d. Show that the sentence above is not entailed by the sentence

$RadioWorks \Rightarrow \neg BatteryDead$

Find a model in which sentence 2 is true and sentence 1 is false, i.e. R, C , O are true, B is false

7. Let *M(x)* be true if x is a mail carrier, *B(x)* is true if x lives in Berkeley, and *K(x,y)* be true if x knows y. Translate the following into FOL:

a. There are at least two mail carriers who live in Berkeley.

$\exists x, y\ M(x) \wedge M(y) \wedge B(x) \wedge B(y) \wedge \neg(x = y)$

b. All the mail carriers who live in Berkeley know each other.

$\forall x, y\ M(x) \wedge M(y) \wedge B(x) \wedge B(y) \Rightarrow K(x,y)$

8. Consider the following sentence:

$((Food \Rightarrow Party) \vee (Drinks \Rightarrow Party)) \Rightarrow ((Food \wedge Drinks) \Rightarrow Party)$

a. Determine, using enumeration, whether the sentence is valid, satisfiable or unsatisfiable. **Valid**

b. Convert the left and right hand sides of the main implication to CNF and verify your answer to a.

$\neg F \vee \neg D \vee P$ for LHS **and same for RHS. Thus this is like** $Q \Rightarrow Q$ **which is valid**

c. Use resolution to prove a.

Negate and convert to CNF:

$$\neg\big((Food \Rightarrow Party) \vee (Drinks \Rightarrow Party) \Rightarrow \big((Food \wedge Drinks) \Rightarrow Party\big)\big)$$

$$= ((F \Rightarrow P) \vee (D \Rightarrow P)) \wedge \neg(F \wedge D \Rightarrow P)$$

$$= (\neg F \vee \neg D \vee P) \wedge (F) \wedge (D) \wedge (\neg P)$$ **which resolves to empty clause, thus the original sentence is valid**

9. Correct the following FOL translations as necessary:

a. Any apartment in Berkeley has lower rent than some apartments in Palo Alto.

$$\forall x \big(Apt(x) \wedge In(x, Berkeley)\big) \Rightarrow \exists y \big((Apt(y) \wedge In(x, PaloAlto)) \Rightarrow < (Rnt(x), Rnt(y))\big)$$

Incorrect, should be

$$\forall x \big(Apt(x) \wedge In(x, Berkeley)\big) \Rightarrow \exists y \big((Apt(y) \wedge In(y, PaloAlto)) \wedge < (Rnt(x), Rnt(y))\big)$$

b. There is exactly one apartment in Palo Alto with rent below $1000.

$$\exists x\ Apt(x) \wedge In(x, PaloAlto) \wedge \forall y \left(Apt(y) \wedge In(y, PaloAlto) \wedge < \big(Rnt(y), Dollars(1000)\big)\right) \Rightarrow y = x$$

Incorrect, should be

$$\exists x\ Apt(x) \wedge In(x, PaloAlto) \wedge < (Rnt(x), Dollars(1000)) \wedge \Big(\forall y \left(Apt(y) \wedge In(y, PaloAlto) \wedge < \big(Rnt($$

c. If an apartment is more expensive than all apartments in Berkeley, it must be in San Francisco

$$\forall x\ Apt(x) \wedge \big(\forall y\ Apt(y) \wedge In(y, Berkeley) \wedge > \big(Rnt(x), Rnt(y)\big)\big) \Rightarrow In(x, SanFrancisco)$$

Incorrect, should be

$$\forall x\ Apt(x) \wedge \big(\forall y\ Apt(y) \wedge In(y, Berkeley) \Rightarrow > \big(Rnt(x), Rnt(y)\big)\big) \Rightarrow In(x, SanFrancisco)$$